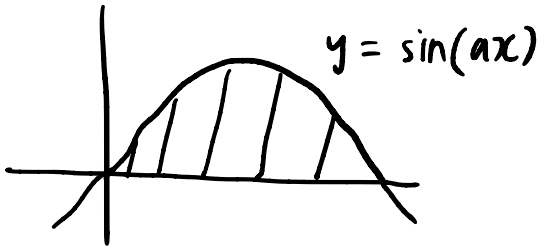
Compute the volume of the solid that results from revolving one hump of $y = \sin(ax)$ around the x-axis.



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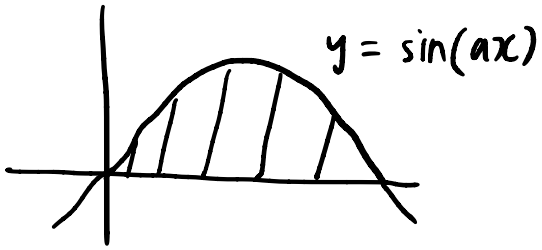

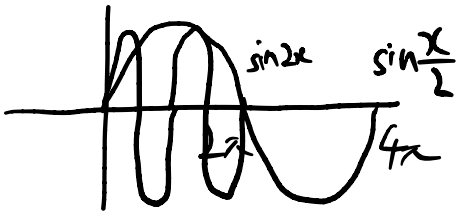

Volume

$$= \int_0^{\frac{\pi}{a}} \pi y^2 \, dx$$

$$= \int_0^{\frac{\pi}{a}} \pi \sin^2 ax \, dx$$

$$= \pi \int_0^{\frac{\pi}{a}} \frac{1 - \cos 2ax}{2} \, dx$$

$$= \frac{\pi}{2} \left(x - \frac{\sin 2ax}{2a} \right) \Bigg|_0^{\frac{\pi}{a}}$$

$$= \frac{\pi}{2} \left(\frac{\pi}{a} - \frac{\sin 2\pi}{2a} \right)$$

$$= \frac{\pi}{2} \left(\frac{\pi}{a} - 0 \right)$$

$$= \frac{\pi^2}{2a}$$

$u = 2ax$

$du = 2a \, dx$

$x = \frac{\pi}{a} \Rightarrow u = 2\pi$

$x = 0 \Rightarrow u = 0$

Joel substituted for $u = ax$ before using the half angle formula and integrating.